Errata for Sierksma and Zwols, 'Linear and Integer Optimization: Theory and Practice', Third Edition

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With many thanks to Ercan Atam for pointing out several of the mistakes listed in this document.

Chapter 1

- Page 4: In the paragraph below the equations for $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, the sentence 'thereby moving the line in Figure 1.2 corresponding to (1.3) to the right' in parentheses should refer to (1.2), not (1.3).
- Page 4: In the second to last sentence on the page, the level line '$3x_1 + 2x_2 = 4\frac{1}{2}$' should be '$3x_1 + 2x_2 = 22\frac{1}{2}$'.
- Page 6: In the last paragraph, 'constraints (1.1) and (1.1)' should read 'constraints (1.1) and (1.2)'.
- Page 17: Model (1.13) should have nonnegativity constraints '$u_1, u_2, x_2 \geq 0$'. (There is no x_3.)
- Page 18: In the second to last paragraph, the first two occurrences of '(1.13)' should read '(1.14)'.
- Page 37: In the paragraph below model (1.22), the interpretation of u_s is slightly wrong. It should say: 'Note that u_s measure the difference between the rate of return of the portfolio in scenario s and the expected rate of return of the portfolio'.
- Page 38: In the paragraph below the equations for the optimal solution, '30%' should read '19%'.

Chapter 2

- Page 58: The proof of Theorem 2.1.2. (bounded case) is in Appendix D.3, on page 606, not in Section 2.1.3 as claimed.
- Page 64: In the first two sentences on the page, C and R should be switched.
- Page 72: The last paragraph in italic should read:

  So, $\mathbf{B}$ is equivalent to a block diagonal matrix (see Appendix B.3), the blocks of which are $\begin{bmatrix}3\end{bmatrix}$ (marked in bold in the matrix above) and an identity matrix. Since the matrix $\begin{bmatrix}3\end{bmatrix}$ and any identity matrix are both trivially invertible, it follows that the matrix $\mathbf{B}$ is invertible; see also Appendix B.7. By a similar argument, $\bar{\mathbf{B}}$ is invertible as well.
- Page 79, Exercise 2.3.1: the formulation of the exercise is slightly confusing. The following is clearer:

(a) Give an example of a two-dimensional LO-model for which (i) the feasible region has a degenerate vertex and (ii) no constraint is redundant with respect to the feasible region.

(b) Give an example of a three-dimensional LO-model for which (i) optimal solution is degenerate, (ii) no technology constraint is redundant with respect to the feasible region, and (iii) all nonnegativity constraints are redundant with respect to the feasible region.

Chapter 3

- Page 91: In the paragraph below the equations for $\mathbf{B}^{-1}\mathbf{N}$ and $\mathbf{B}^{-1}\mathbf{b}$, '(M3)' should be '(M2)'.
- Page 95: In the sentence before Algorithm 3.3.1, 'Section 3.3.1' should read 'Section 3.6'.
- Page 109: The bottom-right entry of the simplex tableau corresponding to $\{1, 5\}$ should be 0, not 1.
- Page 135: In the last sentence of the example, 'Section 1.1.2' should read 'Section 1.2.3'.
- Page 145, Exercise 3.10.3(c): Should read: 'Show that the simplex algorithm **may cycle** ...'
- Page 145, Exercise 3.10.6: Remove the word 'only' (three times).
- Page 147, Exercise 3.10.10: Remove '$(\times 100{,}000)$'. Exchange the 'small' and 'large' columns in the table. The word 'unlimited' should be replaced by 'practically unlimited'.

Chapter 4

- Page 152: 'The new company' should read 'A new company'.
- Page 159, Example 4.2.1: The two dual models (i.e., the third and fourth model in the example) should be **maximizing** models.
- Page 169, Proof of Theorem 4.3.3: The proof of the theorem has several typos. A revised version is included at the end of this document.
- Page 178: Inequality (4.8) should be written as $\mathbf{b}_{NI^c} - (\overline{\mathbf{B}}^{-1}\mathbf{N})^\mathsf{T}\mathbf{b}_{BI^c} \geq \mathbf{0}$. Note that the inequality printed in the book is correct, but the later comment that the left hand side vectors of (4.5) and (4.8) are equal is incorrect.
- Page 186: The paragraph after Example 4.7.1 has typos ($\bar{c}_\beta$ should be $\bar{c}_\alpha$), but also unnecessarily introduces the notation $\bar{c}_k$. It should be replaced by:

The expression (4.12) is called the *dual minimum-ratio test*. This test guarantees that, after pivoting on the (β, NI_α)-entry of the simplex tableau, all objective coefficients remain nonpositive. To see this, we need to show that, after the pivot step, the new objective coefficient corresponding to x_{NI_k} (for any $(k \in \{1, \ldots, n\})$ remains nonpositive, i.e.:

$$(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_k - \frac{(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_\alpha}{(\mathbf{B}^{-1}\mathbf{N})_{\beta,\alpha}}(\mathbf{B}^{-1}\mathbf{N})_{\beta,k} \leq 0. \qquad (4.13)$$

Note that $(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_\alpha \leq 0$ because we started with a simplex tableau in which the objective coefficients are nonpositive, and $(\mathbf{B}^{-1}\mathbf{N})_{\beta,\alpha} < 0$ by the choice of α and β. Hence,

$$\frac{(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_\alpha}{(\mathbf{B}^{-1}\mathbf{N})_{\beta,\alpha}} \geq 0.$$

So, if $(\mathbf{B}^{-1}\mathbf{N})_{\beta,k} \geq 0$, then (4.13) holds. If $(\mathbf{B}^{-1}\mathbf{N})_{\beta,k} < 0$, then (4.13) is equivalent to

$$\frac{(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_k}{(\mathbf{B}^{-1}\mathbf{N})_{\beta,k}} \geq \frac{(\mathbf{c}_{NI}^{\mathsf{T}} - \mathbf{c}_{BI}^{\mathsf{T}}\mathbf{B}^{-1}\mathbf{N})_\alpha}{(\mathbf{B}^{-1}\mathbf{N})_{\beta,\alpha}},$$

which follows immediately from (4.12).

- Page 191, Exercise 4.8.3: the model should be a **minimizing** model. (a) should read 'Formulate and solve the dual model'.
- Page 191, Exercise 4.8.4: the model should be a **minimizing** model.
- Page 192, Exercise 4.8.10: $\mathbf{x}$ and $\mathbf{y}$ should read $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, respectively.

Chapter 5

- Page 196: In the first line after the inequalities, remove the word 'feasible'.
- Page 203: In the first line in the paragraph above (5.4), 'the value' should read 'the objective coefficient'.
- Page 204: The inequalities in the definitions of $\delta_{\min}$ and $\delta_{\max}$ should be reversed.
- Page 215: In the last line on the page, $1\frac{1}{2}$ should be $-1\frac{1}{2}$.
- Page 223, Theorem 5.6.1: The right column in (a) should read 'All optimal solutions degenerate'.
- Page 226, Example 5.6.3: This example may be confusing. The first sentence refers to Example 5.6.5 which is later in the book, and the figure corresponding to it is on page 222.
- Page 231, Example 5.6.5: Figure 5.11 is on page 222, for no apparent reason.
- Page 241, Exercise 5.8.14: the LO-model is in Example 5.6.3.

- Page 242, Exercise 5.8.17: the LO-model for the diet problem is in Section 1.6.1.
- Page 244, Exercise 5.8.23d: This question should read 'How much can the sales prices of the four products vary without influencing the optimal production schedule?'.
- Page 245, Exercise 5.8.24: Raw material **3** turns out to be relative expensive. 'With what mount' should read 'By what amount'.
- Page 245, Exercise 5.8.26: The first sentence below the LO-model should read 'Draw the simplex adjacency graph of this LO-model.' The second sentence should start with 'For each **optimal** basic solution'.

Chapter 7

- Page 328, Exercise 7.5.23: assume that $h_i > 0$ for all i.

Chapter 8

- Page 386, Exercise 8.4.5: this exercise should refer to Section 8.2.1, not 4.3.3.
- Page 391, Exercise 8.4.25: the break between trips should be counted as work time. In (b), the reader should calculate the optimal solution. In (c), the bus drivers must have a **fifteen**-minute break, not thirty (otherwise, the model is infeasible, because both morning trips 4 and 5 can only be combined with afternoon trip 4).

Chapter 12

- Page 443: '$\mathrm{Var}(d_{t+1})$' should read '$\sqrt{\mathrm{Var}(d_{t+1})}$'.

Chapter 14

- Page 491, Exercise 14.9.6: 'z' should be 'z^*', and 'w' should be 'w^*'.
- Page 491, Exercise 14.9.7: 'x_3' should be 'x_3^*', and 'z^+' should be '$(z^+)^*$'.

Chapter 15

- Page 509, Exercise 15.6.3: In the subscript of the summation in the definition of $m_k(\mathbf{a})$, 'a'' should be '$\mathbf{a}'$'.
- Page 510, Exercise 15.6.5: 'the same point' should read 'the same **Owen** point'.
- Page 511, Exercise 15.6.11: You should take $\mathbf{a} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\mathsf{T}$.

Chapter 17

- Page 544, Exercise 17.10.3: The words ‘optimal’ should be removed. (The statement is not true otherwise.)

Author Index

- Page 645: ‘Kuhn, K.W.’ should be ‘Kuhn, H.W.’

The proof of Theorem 4.3.3

The proof of Theorem 4.3.3 (The Strong complementary slackness theorem) has several typos. Below is a revised version, with a few additional clarifications.

Theorem 4.3.3. (*Strong complementary slackness theorem*)
For any standard LO-model that has an optimal solution, there exist an optimal primal solution $\mathbf{x}^* = \begin{bmatrix} x_1^* & \dots & x_n^* \end{bmatrix}^\mathsf{T}$ and an optimal dual solution $\mathbf{y}^* = \begin{bmatrix} y_1^* & \dots & y_m^* \end{bmatrix}^\mathsf{T}$ such that every pair of complementary dual variables (x_i^*, y_j^*) has the property that exactly one of x_i^* and y_j^* is nonzero.

Proof of Theorem 4.3.3. Let the standard primal model $\max\left\{\mathbf{c}^\mathsf{T}\mathbf{x} \;\middle|\; \mathbf{A}\mathbf{x} \le \mathbf{b}, \mathbf{x} \ge \mathbf{0}\right\}$ and its dual model $\min\left\{\mathbf{b}^\mathsf{T}\mathbf{y} \;\middle|\; \mathbf{A}^\mathsf{T}\mathbf{y} \ge \mathbf{c}, \mathbf{y} \ge \mathbf{0}\right\}$ be denoted by (P) and (D), respectively. We first prove the following weaker statement.

($\star$) For each $i = 1, \dots, n$, either there exists an optimal solution $\mathbf{x}^* = \begin{bmatrix} x_1^* & \dots & x_n^* \end{bmatrix}^\mathsf{T}$ of (P) such that $x_i^* > 0$, or there exists an optimal solution $\mathbf{y}^* = \begin{bmatrix} y_1^* & \dots & y_m^* \end{bmatrix}^\mathsf{T}$ of (D) such that $(\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c})_i > 0$.

To prove ($\star$), let z^* be the optimal objective value of (P). Let $i \in \{1, \dots, n\}$, and consider the LO-model:

$$\begin{aligned} \max \quad & \mathbf{e}_i^\mathsf{T}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & -\mathbf{c}^\mathsf{T}\mathbf{x} \le -z^* \\ & \mathbf{x} \ge \mathbf{0}. \end{aligned} \tag{P'}$$

(Here, $\mathbf{e}_i$ is the i'th unit vector in $\mathbb{R}^n$.) Model (P') is feasible, because any optimal solution of (P) is a feasible solution of (P'). Let z' be the optimal objective value of (P'). Clearly, $z' \ge 0$. We now distinguish two cases.

- Case 1: $z' > 0$. Let $\mathbf{x}^*$ be an optimal solution of (P'). Since $\mathbf{A}\mathbf{x}^* \le \mathbf{b}$, $\mathbf{x}^* \ge \mathbf{0}$, and $\mathbf{c}^\mathsf{T}\mathbf{x}^* \ge z^*$, it follows that $\mathbf{x}^*$ is an optimal solution of (P). Moreover, $x_i^* = \mathbf{e}_i^\mathsf{T}\mathbf{x}^* = z' > 0$. Hence, ($\star$) holds.

- Case 2: $z' = 0$. Since (P') has optimal objective value 0, it follows from Theorem 4.2.4 that the dual of (P') also has optimal objective value 0:

$$\begin{aligned} \min \quad & \mathbf{b}^\mathsf{T}\mathbf{y} - z^*\lambda \\ \text{s.t.} \quad & \mathbf{A}^\mathsf{T}\mathbf{y} - \lambda\mathbf{c} \ge \mathbf{e}_i \\ & \mathbf{y} \ge \mathbf{0}, \lambda \ge 0. \end{aligned} \tag{D'}$$

  Let $(\mathbf{y}', \lambda')$ be an optimal solution of (D'). Note that $\mathbf{b}^\mathsf{T}\mathbf{y}' = z^*\lambda'$. We consider two subcases:

- Case 2a: $\lambda' = 0$. Then, we have that $\mathbf{b}^\mathsf{T}\mathbf{y}' = 0$ and $\mathbf{A}^\mathsf{T}\mathbf{y}' \geq \mathbf{e}_i$. Let $\hat{\mathbf{y}}$ be an optimal solution of (D). It satisfies $\mathbf{b}^\mathsf{T}\hat{\mathbf{y}} = z^*$. Then, $\mathbf{y}^* = \hat{\mathbf{y}} + \mathbf{y}'$ satisfies $\mathbf{y}^* = \hat{\mathbf{y}} + \mathbf{y}' \geq \mathbf{0}$, $\mathbf{b}^\mathsf{T}\mathbf{y}^* = \mathbf{b}^\mathsf{T}(\hat{\mathbf{y}} + \mathbf{y}') = \mathbf{b}^\mathsf{T}\hat{\mathbf{y}} = z^*$, and $\mathbf{A}^\mathsf{T}\mathbf{y}^* = \mathbf{A}^\mathsf{T}(\hat{\mathbf{y}} + \mathbf{y}') \geq \mathbf{c} + \mathbf{e}_i \geq \mathbf{c}$. Hence, $\mathbf{y}^*$ is an optimal solution of (D), and it satisfies $(\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c})_i = (\mathbf{A}^\mathsf{T}(\hat{\mathbf{y}} + \mathbf{y}') - \mathbf{c})_i \geq (\mathbf{e}_i)_i = 1 > 0$, and so $(\star)$ holds.
- Case 2b: $\lambda' > 0$. Define $\mathbf{y}^* = \mathbf{y}'/\lambda'$. Then, we have that $\mathbf{y}^* \geq \mathbf{0}$, $\mathbf{b}^\mathsf{T}\mathbf{y}^* = (\mathbf{b}^\mathsf{T}\mathbf{y}')/\lambda' = (z^*\lambda')/\lambda' = z^*$, and $\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c} = (\mathbf{A}^\mathsf{T}\mathbf{y}' - \lambda'\mathbf{c})/\lambda' \geq \mathbf{e}_i/\lambda'$. Hence, $\mathbf{y}^*$ is an optimal solution of (D), and it satisfies $(\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c})_i = (\mathbf{e}_i/\lambda')_i = 1/\lambda' > 0$, and so $(\star)$ holds.

We can now prove the theorem. For $i = 1, \ldots, n$, apply statement $(\star)$ to (P) and (D) to obtain $\mathbf{x}^{(i)}$ and $\mathbf{y}^{(i)}$ such that: (1) $\mathbf{x}^{(i)}$ is an optimal solution of (P), (2) $\mathbf{y}^{(i)}$ is an optimal solution of (D), and (3) either $x_i^{(i)} > 0$ or $(\mathbf{A}^\mathsf{T}\mathbf{y}^{(i)} - \mathbf{c})_i > 0$. Similarly, for $j = 1, \ldots, m$, apply statement $(\star)$ to (D) and (P) (i.e., with the primal and dual models switched) to obtain $\mathbf{x}^{(n+j)}$ and $\mathbf{y}^{(n+j)}$ such that: (1) $\mathbf{x}^{(n+j)}$ is an optimal solution of (P), (2) $\mathbf{y}^{(n+j)}$ is an optimal solution of (D), and (3) either $y_j^{(n+j)} > 0$ or $(\mathbf{b} - \mathbf{A}\mathbf{x}^{(n+j)})_j > 0$. Define:

$$\mathbf{x}^* = \frac{1}{n+m}\sum_{k=1}^{n+m}\mathbf{x}^{(k)}, \text{ and } \mathbf{y}^* = \frac{1}{n+m}\sum_{k=1}^{n+m}\mathbf{y}^{(k)}.$$

Due to Theorem 3.7.1 and the fact that $\mathbf{x}^*$ is a convex combination of optimal solutions of (P) (why?), we have that $\mathbf{x}^*$ is also a (not necessarily basic) optimal solution of (P). Similarly, $\mathbf{y}^*$ is an optimal solution of (D). Hence, by Theorem 4.3.1, it follows that $\mathbf{x}^*$ and $\mathbf{y}^*$ satisfy the complementary slackness relations.

It remains to show that $\mathbf{x}^*$ and $\mathbf{y}^*$ satisfy the assertions of the theorem. To do so, consider any pair of complementary dual variables (x_i^*, y_{m+i}^*) with $i = 1, \ldots, n$. We have that:

$$x_i^* = \left(\frac{1}{n+m}\sum_{k=1}^{n+m}\mathbf{x}^{(k)}\right)_i = \frac{1}{n+m}\sum_{k=1}^{n+m}x_i^{(k)},$$

and

$$y_{m+i}^* = (\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c})_i = \left(\mathbf{A}^\mathsf{T}\left(\frac{1}{n+m}\sum_{k=1}^{n+m}\mathbf{y}^{(k)}\right) - \mathbf{c}\right)_i = \frac{1}{n+m}\sum_{k=1}^{n+m}(\mathbf{A}^\mathsf{T}\mathbf{y}^{(k)} - \mathbf{c})_i.$$

Note that $x_i^{(k)} \geq 0$ and $(\mathbf{A}^\mathsf{T}\mathbf{y}^{(k)} - \mathbf{c})_i \geq 0$ for $k = 1, \ldots, n+m$. Moreover, by the construction of $\mathbf{x}^{(i)}$ and $\mathbf{y}^{(i)}$, we have that either $x_i^{(i)} > 0$ or $(\mathbf{A}^\mathsf{T}\mathbf{y}^{(i)} - \mathbf{c})_i > 0$. Therefore, either $x_i^* > 0$ or $(\mathbf{A}^\mathsf{T}\mathbf{y}^* - \mathbf{c})_i > 0$ (and not both, because $\mathbf{x}^*$ and $\mathbf{y}^*$ satisfy the complementary slackness relations). It is left to the reader that the conclusion also holds for any pair of complementary dual variables (x_{n+j}^*, y_j^*) with $j = 1, \ldots, m$. □